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An Abatement Investment Strategy with Stochastic Abatement Technology*

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1 Introduction

In this paper, we investigate a pollutant abatement investment strategy when the abatement technology is stochastic. We consider a production economy and develop a stochastic endogenous growth model. For analytical simplicity, the economy consists of a representative consumer and a firm. The representative consumer has constant relative risk-averse preferences and maximizes his/her utility. The representative firm produces output using production capital and maximizes its profit. The production process, however, generates pollutant emissions proportional to output, and these damage the consumer. Therefore, the firm must invest in pollutant abatement activities to reduce pollutant emissions. We assume that the level of abatement technology is governed by a stochastic differential equation. We formulate both agents' problems as a central planner's problem that maximizes social welfare and obtain a nonlinear partial differential equation that derives the optimal investment strategy.

Smulders and Gradus (1996), Steger (2005), Wälde (2011), and Bucci et al. (2011) also investigated the social welfare maximizing problem. Smulders and Gradus investigated the sustainable economic growth rate when pollutants from a production process were included in the economy, and showed the socially optimal growth rate. Steger, Wälde, and Bucci et al. considered stochastic technological progress and investigated an endogenous stochastic growth model. This paper considers both components, pollutants from the production process and stochastic technology, and investigates the optimal investment strategy for production capital and abatement activity when the abatement technology is stochastic. The rest of the paper is organized as follows. In Section 2, we analyze the social welfare maximizing problem, excluding pollution from the production process as a base case. We derive the optimal consumption rate in the closed form. In Section 3, we extend the base case model of Section 2 by incorporating pollutants from the production process into the model. We solve the social welfare maximizing problem and derive the equation, which leads to the optimal abatement investment strategy. Finally, Section 4 concludes the paper.

2 Base Case Model

In this section, we consider a social maximizing problem, excluding pollutants from the production process, as a base case model.

We consider a production economy with an infinite time horizon. For analytical simplicity, the economy consists of a representative consumer and a firm.

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The representative firm produces output Y_t using production capital K. The firm's production function $F(K_t)$ is given by the following AK-form:

$$Y_t = F(K_t) = AK_t,$$

where A is the level of production technology. As in Wälde (2011), the dynamics of the capital stock are given by:

$$\mathrm{d}K_t = (I_t - \delta K_t)\mathrm{d}t - \sigma_K K_t \mathrm{d}W_t^K, \qquad K_0 = k \tag{2.1}$$

where I_t is the capital investment, $\delta \in (0, 1)$ is the depreciation rate, $\sigma > 0$ is the volatility, and W_t^K is a standard Brownian motion on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\}_{t \geq 0})$.

The representative consumer receives utility from consumption C_t. He/she has constant relative risk-averse preferences and a utility function given by:

$$U(C_t) = \frac{1}{1-\gamma} C_t^{1-\gamma}, \tag{2.2}$$

where $\gamma > 0$ is the degree of relative risk aversion. The consumer's budget constraint is:

$$Y_t = I_t + C_t.$$

Then, we have:

$$I_t = Y_t - C_t \tag{2.3}$$

It follows from (2.1) and (2.3) that the dynamics of the production capital can be rewritten as:

$$\mathrm{d}K_t = (Y_t - \delta K_t - C_t)\mathrm{d}t - \sigma_K K_t \mathrm{d}W_t^K, \qquad K_0 = k \tag{2.4}$$

The representative firm maximizes its profits. The representative consumer maximizes his/her utility, subject to the budget constraint. Therefore, the central planner's problem is to choose consumption $\{C_t\}$ in order to maximize the social welfare:

$$\hat{V}(k) = \max_{\{C_t\}} \mathbb{E}\left[\int_0^\infty \mathrm{e}^{-rt} U(C_t)\mathrm{d}t\right]. \tag{2.5}$$

The Hamilton-Jacobi-Bellman (HJB) equation of the central planner's problem (2.5) is:

$$r\hat{V}(k) = \max_c \left\{ \frac{1}{1-\gamma} c^{1-\gamma} + (Ak - \delta k - c)\hat{V}_k(k) + \frac{1}{2}\sigma_K^2 k^2 \hat{V}_{kk}(k) \right\}. \tag{2.6}$$

From the first-order condition for the optimality, we obtain the optimal consumption as follows:

$$\hat{c}^* = \hat{V}_k(k)^{-\frac{1}{\gamma}}. \tag{2.7}$$

From the utility function (2.2), we can guess that a candidate solution to the value function is:

$$\hat{V}(k) = Bk^{1-\gamma}, \tag{2.8}$$

where B is a constant to be determined. Substituting (2.8) into (2.6), we obtain:

$$\left[\gamma(1-\gamma)^{-\frac{1}{\gamma}} B^{-\frac{1}{\gamma}} + (A - \delta)(1-\gamma) - \frac{\sigma_K^2}{2}(1-\gamma)\gamma - r\right] Bk^{1-\gamma} = 0. \tag{2.9}$$

As (2.9) must hold for all k, the constant to be determined, B, is:

$$B = \gamma^{\gamma}(1-\gamma)^{-1}\left[r - (A-\delta)(1-\gamma) + \frac{\sigma^2}{2}\gamma(1-\gamma)\right]. \tag{2.10}$$

Thus, we obtain the optimal consumption in the following analytical form:

$$\hat{c}^* = \gamma^{-1}\left[r - (A-\delta)(1-\gamma) + \frac{\sigma^2}{2}\gamma(1-\gamma)\right]k. \tag{2.11}$$

3 The Model with Pollution

In this section, we extend the base case model by incorporating pollutants from the production process.

For simplicity, in this section, we assume that the production capital stock grows deterministically and is given by:

$$\mathrm{d}K_t = (I_t - \delta K_t)\mathrm{d}t, \qquad K_0 = k. \tag{3.1}$$

The output production process generates pollutant emissions proportional to the output level, $\eta F(K_t)$, where $\eta > 0$ is the emission conversion coefficient. As the pollutant damages the consumer, the firm invests in pollutant abatement activity $H(I_t^A)$:

$$H(I_t^A) = X_t(I_t^A)^2, \tag{3.2}$$

where I_t^A is the abatement investment and X_t is the level of abatement technology. As in Steger (2005) and Wälde (2011), we assume that the abatement technology is governed by following the geometric Brownian motion:

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}W_t, \qquad X_0 = x. \tag{3.3}$$

The abatement activity reduces the pollutant emissions, and the net pollutant emissions E_t are expressed as:

$$E_t = \eta F(K_t) - H(I_t^A). \tag{3.4}$$

The pollutant emissions are accumulated by:

$$\mathrm{d}P_t = (E_t - \delta_P P_t)\mathrm{d}t, \qquad P_0 = p, \tag{3.5}$$

where $\delta_P \in (0,1)$ is the depreciation rate of the pollutant stock P_t.

The representative consumer suffers from the pollutant stock. As in Smulders and Gradus (1996), incorporating disutility from the pollutant, the consumer's utility function becomes:

$$U(C_t, P_t) = \frac{1}{1-\gamma}\left(C_t P_t^{-\phi}\right)^{1-\gamma}, \tag{3.6}$$

where $\phi > 0$ is the disutility coefficient. The budget constraint of the consumer is:

$$\begin{aligned} Y_t &= I_t + I_t^A + C_t \\ &= I_t + \theta_t Y_t + C_t, \end{aligned}$$

where $\theta_t > 0$ is the abatement investment share, which is given by:

$$\theta_t = \frac{I_t^A}{Y_t}.$$

By employing this share, the net emissions flow is calculated as:

$$E_t = \eta AK_t - X_t\theta_t^2(AK_t)^2 \tag{3.7}$$

Substituting (3.7) into (3.5), the dynamics of the pollutant stock can be rewritten as:

$$\mathrm{d}P_t = [\eta AK_t - X_t\theta_t^2(AK_t)^2 - \delta_P P_t]\mathrm{d}t, \qquad P_0 = p. \tag{3.8}$$

Rewriting the budget constraint of the consumer, the capital investment is:

$$I_t = (1-\theta)Y_t - C_t. \tag{3.9}$$

From (3.1) and (3.9), the dynamics of the capital stock are rewritten as:

$$\mathrm{d}K_t = ((1-\theta)Y_t - \delta K_t - C_t)\mathrm{d}t, \qquad K_0 = k. \tag{3.10}$$

As in Section 2, the representative firm maximizes its profits and the representative consumer maximizes his/her utility, subject to the budget constraint. However, the firm's production activity generates a pollutant as a by-product, and the consumer suffers from the pollutant, which reduces his/her utility. Therefore, the central planner's problem is to choose a consumption level and an investment share for the abatement activity in order to maximize social welfare:

$$V(k,p,x) = \max_{\{C_t,\theta_t\}} \mathbb{E}\left[\int_0^\infty \mathrm{e}^{-rt}(U(C_t,P_t))\mathrm{d}t\right]. \tag{3.11}$$

The HJB equation of the central planner's problem (3.11) is:

$$\begin{aligned} rV = \max_{c,\theta}\Bigg\{ & \frac{1}{1-\gamma}(cp^{-\phi})^{1-\gamma} + [(1-\theta)Ak - \delta k - c]V_k + \\ & [(\eta Ak - x\theta^2(Ak)^2 - \delta_P p]V_p + \mu x V_x + \frac{1}{2}\sigma^2x^2V_{xx}\Bigg\}. \end{aligned} \tag{3.12}$$

From the first-order condition for the optimality, we obtain the optimal consumption c^* and optimal abatement investment share θ^*:

$$c^* = p^{-\frac{\phi(1-\gamma)}{\gamma}}V_k^{-\frac{1}{\gamma}}, \tag{3.13}$$

$$\theta^* = -\frac{1}{2}x^{-1}(Ak)^{-1}\frac{V_k}{V_p}. \tag{3.14}$$

Substituting (3.13) and (3.14) into (3.12), we obtain the following nonlinear partial differential equation:

$$\begin{aligned} rV = & \frac{\gamma}{1-\gamma}p^{-\frac{\phi(1-\gamma)}{\gamma}}V_k^{-\frac{1-\gamma}{\gamma}} + \left[(A-\delta)k + \frac{1}{2}x^{-1}(Ak)^{-1}\frac{V_k}{V_p} - \frac{1}{4}x^{-1}V_k\right]V_k \\ & + [\eta AkV_p - \delta_P p]V_p + \mu x V_x + \frac{1}{2}\sigma^2x^2V_{xx}. \end{aligned} \tag{3.15}$$

The optimal consumption level and abatement investment share are derived from the nonlinear partial differential equation (3.15). Because of the nonlinearity, we have to solve the equation (3.15) numerically.

4 Conclusion

In this paper, we analyzed a pollutant abatement investment strategy when the abatement technology is stochastic. We obtained the nonlinear partial differential equation, which derives the optimal abatement investment strategy. Because the partial differential equation is nonlinear, it is solved numerically. We leave the numerical calculation for future work.

There are several ways to extend this paper in future. For instance, we could consider abatement technological progress that follows a jump diffusion process, and we could incorporate both the production and the abatement technology into the model.

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